

New Scaling in High Energy DIS.

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Abstract

We develop a new approach for solving the non-linear evolution equation in the low x_B region and show that the remarkable “geometric” scaling of its solution holds not only in the saturation region, but in much wider kinematical region. This is in a full agreement with experimental data (Golec-Biernat, Kwiecinski and Stasto).

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1 Introduction.

We believe that unitarity holds for any physical process. At very high energies it manifests itself as a suppression of growth of cross sections as a function of energy. While at moderate energies the linear evolution equations hold, at higher energy corrections to those equations arise which have essentially non-linear form. It was suggested in [1, 2, 3, 4] that there exists a certain scale, called the saturation scale $Q_s^2(x_B)$, at which those non-linear corrections set in. This scale characterizes the high density phase of QCD which is non-perturbative despite the smallness of the QCD coupling constant. Consider the total cross section for deeply inelastic scattering of virtual photon off the target $\sigma(Q^2, x_B)$. In the kinematical region $Q^2 \gg Q_s^2(x_B) > \Lambda^2$, where Λ is a non-perturbative scale, the DGLAP evolution equations describe the experimental data very well. In the infinite momentum frame, photon interacts with only one parton in the partonic cascade. On the other hand, we expect that in the kinematical region $Q^2 < Q_s^2(x_B)$ (high density region) the virtual photon will most probably interact simultaneously with at least two partons. Equation which takes into account the possible simultaneous interactions of the photon with two partons was derived in [1, 3] in double logarithmic approximation and is written in parton language. More than decade later it was shown by Balitsky[5], that such quadratic interactions describe the parton evolution in the whole kinematical region (in the leading $\ln 1/x_B$) including very low x_B . Recently, this result was independently derived by Kovchegov[6] in the framework of the dipole model[7, 8], by several authors using semi-classical approach[9, 10] and by Braun, using the standard form of the Pomeron-target coupling[11]. In this paper we will consider the non-linear evolution equation written in the dipole model picture.

In the dipole model the deep inelastic scattering of virtual photon off the target has two consequent, well separated in time, stages: decay of the photon into the system of colour dipoles described by the wave function $\Phi(\vec{x}, z)$ and interaction of those dipoles with target with amplitude $N(\vec{x}, y; \vec{b}_t)$, where \vec{x} stands for dipole of size x , b_t is an impact parameter and $y = \ln(x_B^0/x_B)$ is the rapidity defined such that evolution starts at $y = 0$. We will assume that the typical transverse extent of the dipole amplitude is much smaller then the size of the target and the typical impact parameter $x \ll R, b_t$. Then one can significantly simplify the impact parameter dependence of the amplitude[12]. The high parton density evolution equation in the dipole approach now reads[6]

$$\begin{aligned} \frac{\partial N(\vec{x}_{01}^2, y; \vec{b}_t)}{\partial y} &= -\frac{2\alpha_s N_c}{2\pi} \ln\left(\frac{\vec{x}_{01}^2}{\rho^2}\right) N(\vec{x}_{01}^2, y; \vec{b}_t) \\ &+ \frac{\alpha_s N_c}{2\pi} \int_{\rho} d^2 \vec{x}_2 \frac{\vec{x}_{01}^2}{\vec{x}_{02}^2 \vec{x}_{12}^2} \left[2N(\vec{x}_{02}^2, y; \vec{b}_t) - N(\vec{x}_{02}^2, y; \vec{b}_t) N(\vec{x}_{12}^2, y; \vec{b}_t) \right] , \end{aligned} \quad (1)$$

where ρ is ultraviolet cut-off. The kernel of this equation

$$|\Psi(\vec{x}_{01} \rightarrow \vec{x}_{02} + \vec{x}_{12})|^2 = \frac{x_{01}^2}{x_{02}^2 x_{12}^2} ,$$

describes decay of the dipole \vec{x}_{01} into two dipoles \vec{x}_{02} and \vec{x}_{12} . In the limit $N \ll 1$ this equation reduces to the BFKL one[13]. The initial condition is taken to be of the Glauber form[14, 6]. Once $N(\vec{x}, y; \vec{b}_t)$ is known one can calculate the structure function as a convolution of it with the squared photon's wave function [7, 8, 15, 16]

$$F_2(x_B, Q^2) = \frac{Q^2}{4\pi^2 \alpha_{EM}} \int \frac{d^2 \vec{x}}{2\pi} dz |\Phi(\vec{x}, z)|^2 \int d^2 \vec{b}_t N(\vec{x}, y; \vec{b}_t) , \quad (2)$$

where z is a fraction of the photon's energy taken off by a struck parton. Eq. (1) can be written in the momentum space as well. Define, following[12], the two-dimensional Fourier transform

of the amplitude $N(\vec{x}, y)$:

$$N(\vec{x}, y) = \vec{x}^2 \int_0^\infty dk k J_0(kx) \tilde{N}(k, y) \quad (3)$$

$$\tilde{N}(k, y) = \int_0^\infty \frac{dx}{x} J_0(kx) N(x^2, y) , \quad (4)$$

where the fact that neither Eq. (1) nor initial condition (see Ref. [12]) depend on a dipole direction was used to integrate over polar angle explicitly. Then Eq. (1) can be written as

$$\frac{\partial \tilde{N}(k, y)}{\partial y} = \bar{\alpha}_s \hat{\chi}(\hat{\gamma}(k)) \tilde{N}(k, y) - \bar{\alpha}_s \tilde{N}^2(k, y) , \quad (5)$$

where $\hat{\chi}(\hat{\gamma}(k))$ is an operator such that

$$\hat{\gamma}(k) = 1 + \frac{\partial}{\partial \ln k^2} \quad (6)$$

is an operator corresponding to the anomalous dimension of the gluon structure function and the operator $\hat{\chi}$ corresponds to the following function

$$\chi(\gamma) = 2\psi(1) - \psi(1 - \gamma) - \psi(\gamma) \quad (7)$$

which is an eigenvalue of the the BFKL equation.¹ We used convenient notation $\bar{\alpha}_s = \alpha_s N_c / \pi$.

It was pointed out by many authors[4, 17, 18] that in the high density region (defined above) one expects that inclusive observables will show remarkable scaling behaviour, which means that they become a function of only one variable $Q^2/Q_S^2(x_B, b_t)$. It was shown that both GLR and Eq. (1) has this property[17, 18]. In particular, we found in Ref. [18] the scaling solution of Eq. (1) in the saturation region $Q^2 \ll Q_S^2(x_B, b_t)$. However, the experimental verification of this statement is quite difficult for technical reasons. So, it was a great surprise when it turned out that this scaling behaviour (so called “geometric” scaling) holds with 10% accuracy in the *whole* kinematical region $x_B < 0.01$ [20].

The goal of our paper is to show that indeed, the solution of the Eq. (5) scales with good accuracy in a wide high energy region. We will begin by assuming *a priori* that such scaling solution exists. In Sec. 2 we reduce Eq. (5) to the non-linear one-dimensional equation by introducing the scaling variable ξ . We then suggest a model for the kernel χ of Eq. (5) in the saturation and diffusion kinematical regions. In Sec. 3 we solve the one-dimensional (i.e. scaling) equation in the framework of this model. Then, in Sec. 4 we consider scaling-violating corrections to the scaling solution, estimate numerically the size of those corrections and found that they are small in the experimentally accepted high energy kinematical region. Conclusions and discussion are presented in Sec. 5.

2 Definition of the problem.

To proceed we have to specify the critical line $k^2 = k_S^2(x_B)$ at which shadowing corrections set in. In Ref. [18] we found the critical line by matching the double logarithmic solution of Eq. (1) from the kinematical region $\ln k^2 \gg \alpha_s y \sim 1$ (to the right of the critical line in $(\ln k^2, y)$)

¹Note, that χ_{dipole} which was used by A. Mueller in the dipole model and by Yu. Kovchegov in Ref. [6] is different from that defined originally in BFKL papers [13]. The relation between them is follows: $2\chi_{dipole}(\lambda = 2(1 - \gamma)) = \chi_{BFKL}(\gamma) \equiv \chi(\gamma)$. λ corresponds to the operator $\hat{\lambda} = -\frac{\partial}{\partial \ln k}$.

coordinates) with saturating solution from the region $\alpha_s y \gg \ln k^2$ (to the left of the critical line). It reads:

$$4\bar{\alpha}_s y = \ln \frac{k^2}{\Lambda^2} + \beta(b_t, A) \quad , \quad (8)$$

where

$$\beta(b_t, A) = -2 \ln S(b_t, A) - \frac{2}{3} \ln A \quad , \quad (9)$$

$S(b_t, A)$ is target profile function and A is a number of nucleons in the target.

Strictly speaking, Eq. (8) is valid at sufficiently large values of y . Indeed, the Glauber initial condition implies $Q_S^2(y=0) \sim A^{1/3}$ while at large energies $Q_S^2(y) \sim \exp(4\bar{\alpha}_s y) A^{2/3}$. Hence, Eq. (8) holds for $Q_S^2(y) \gg Q_S^2(y=0)$. Throughout this paper we assume that this condition is satisfied².

In the case of DIS on proton the good approximation for $S(b_t)$ is the Gaussian profile function

$$S(b_t) = e^{-\frac{b_t^2}{R_P^2}} \quad . \quad (10)$$

For nuclear target the Woods-Saxon[19] profile function which can be modeled by

$$S(b_t, A) = \theta(R_A - b_t) + \theta(b_t - R_A) e^{-\frac{b_t}{h}} \quad (11)$$

is usually used.

Consequently, let us define the scaling variable

$$\xi = 4\bar{\alpha}_s y - \ln \frac{k^2}{\Lambda^2} - \beta(b_t, A) \quad . \quad (12)$$

It was shown in Ref. [17] that as one approaches the saturation region, scattering amplitude becomes a function of only one variable ξ . Since the scaling variable ξ is defined up to some additive constant we require that at $\xi \geq 0$ the amplitude be a function of only this variable. Hence, at $\xi < 0$ one has to take into account scaling-violating corrections which grow as ξ gets smaller and finally, at some small ξ become of the same order as the scaling solution, thus destroying the scaling behaviour. So, we look for the solution to the Eq. (5) in the following form:

$$\tilde{N}(k, y; b_t) = \tilde{N}(\xi(k, y, b_t)) + \delta\tilde{N}(k, y; b_t) \quad , \quad (13)$$

assuming that scaling-violating correction $\delta\tilde{N}(k, y; b_t)$ is small perturbation of the scaling solution $\tilde{N}(\xi)$ at $\xi < 0$ and vanishes at $\xi \geq 0$. The boundary condition for the correction is

$$\delta\tilde{N}(\xi = 0, y; b_t) = \delta(y) \quad . \quad (14)$$

It was argued in Ref. [17] that anomalous dimension of the amplitude equals $\gamma = \frac{1}{2}$ on the boundary of the kinematical region where the amplitude is a function of only one variable (this boundary is defined as $\xi=0$). This observation provides an initial condition for the scaling solution

$$\frac{d \ln \tilde{N}(\xi)}{d\xi} \Big|_{\xi=0} = \frac{1}{2} \quad . \quad (15)$$

It is convenient to change variables in Eq. (5) $(y, k) \rightarrow (y, \xi)$ which means the following substitutions

$$\frac{\partial}{\partial y} \rightarrow \frac{\partial}{\partial y} + 4\bar{\alpha}_s \frac{\partial}{\partial \xi} \quad ; \quad \frac{\partial}{\partial \ln k^2} \rightarrow -\frac{\partial}{\partial \xi} \quad . \quad (16)$$

²We are going to discuss the A dependence of the critical line at not too large y in a separate publication.

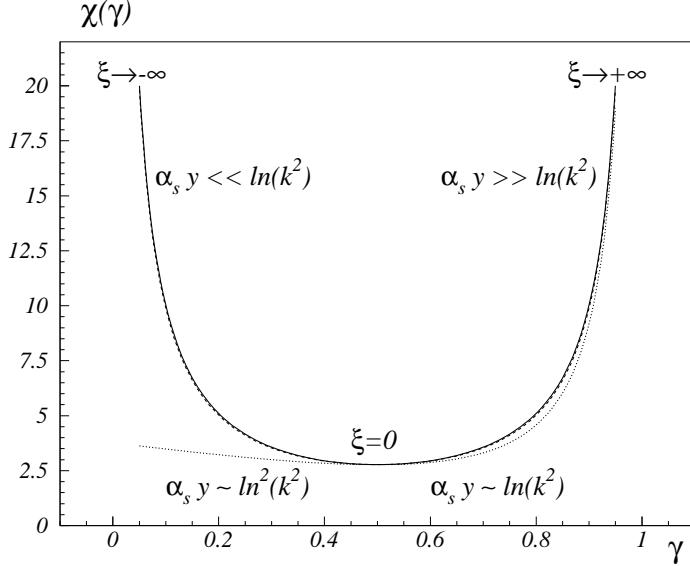


Figure 1: *The eigenvalue of the BFKL evolution equation kernel as a function of the anomalous dimension of the gluon structure function. Solid line is the exact χ as given by Eq. (7), dashed line corresponds to the model Eq. (20) and perfectly fits χ (it is almost indistinguishable from solid line), the dotted line is model for the right branch Eq. (21). The different kinematical regions are shown.*

Using these formulae one casts Eq. (5) to the form

$$\frac{\partial}{\partial y} \tilde{N}(\xi, y; b_t) + 4\bar{\alpha}_s \frac{\partial}{\partial \xi} \tilde{N}(\xi, y; b_t) = \bar{\alpha}_s \chi \left(1 - \frac{\partial}{\partial \xi}\right) \tilde{N}(\xi, y; b_t) - \bar{\alpha}_s \tilde{N}(\xi, y; b_t)^2 \quad (17)$$

2.1 The model for the kernel.

We do not know the exact analytical solution to the Eq. (17) even if we assume that \tilde{N} is a function of only one scaling variable. To simplify this equation we suggest the model for the function $\chi(\gamma)$. Note the following properties of this function which follows from its definition Eq. (7) and definition of the di-gamma function $\psi(\gamma)$:

1. $\chi(\gamma)$ is defined in the region $0 < \gamma < 1$ (see Fig. 1).
 - $\gamma \rightarrow 0$ corresponds to the double logarithmic approximation to the BFKL (or DGLAP) equation, i.e. $\ln k^2 \gg \alpha_s y \sim 1$.
 - $\gamma \rightarrow 1$ corresponds to the saturation region, i.e. $\ln k^2 \ll \alpha_s y \sim 1$.
 - $\gamma \approx \frac{1}{2}$ corresponds to the diffusion approximation, i.e. $\ln^2 k^2 \sim \alpha_s y \sim 1$.
- 2.

$$\chi(\gamma) = \chi(1 - \gamma) \quad (18)$$

- 3.

$$\chi(\gamma) = \frac{1}{\gamma} + 2 \sum_{n=1}^{\infty} \zeta(2n+1)(1-\gamma)^{2n} \quad (19)$$

4. χ has minimum at $\gamma = \frac{1}{2}$, $\chi(\frac{1}{2}) = 4 \ln 2$.

Our model for χ in the whole region $1 < \gamma < 0$ is

$$\chi(\gamma) = \frac{1}{\gamma} + \frac{1}{1-\gamma} + 4 \ln 2 - 4 \quad . \quad (20)$$

It is easily seen that this function satisfies properties 2 and 4. It has also correct asymptotic behaviour at the end points $\gamma \rightarrow 0, 1$. Two last terms in the r.h.s. of Eq. (20) is an even polynom which replaces the even polynom in the r.h.s. of Eq. (19).

In the scaling region $\frac{1}{2} \leq \gamma < 1$ one can expand γ^{-1} term near some point from $[0, 1)$. This gives the term $\sim \gamma$. The model for the right branch of χ , that satisfies property 2 and has correct asymptotic at $\gamma \rightarrow 1$ reads

$$\chi(\gamma) = \frac{1}{1-\gamma} + 4 \ln 2 - 4\gamma \quad . \quad (21)$$

The main assumption of the model is that higher derivatives of the amplitude are much smaller than the amplitude itself. In the next section the scaling solution will be found which justifies our assumption.

In the diffusion region $\xi \approx 0$ we can expand χ near the point $\gamma \approx \frac{1}{2}$

$$\chi(\gamma) = 4 \ln 2 + 14\zeta(3) \left(\gamma - \frac{1}{2}\right)^2 \quad . \quad (22)$$

This is an approximation in which we will calculate $\delta\tilde{N}(\xi, y)$ in Sec. 4.

3 Solution to the scaling equation.

Using Eq. (13) and Eq. (21) in Eq. (17) we get equation for the scaling amplitude $\tilde{N}(\xi)$ at $\xi \geq 0$

$$\tilde{N}(\xi) - 4(1 - \ln 2)\tilde{N}'(\xi) - 2\tilde{N}(\xi)\tilde{N}'(\xi) = 0 \quad . \quad (23)$$

Integration of this equation yields

$$4(1 - \ln 2) \ln \tilde{N} + 2\tilde{N} = \xi - \xi_0 \quad . \quad (24)$$

To find the value of the integration constant ξ_0 we rewrite Eq. (23) in the form

$$\frac{d \ln \tilde{N}(\xi)}{d\xi} = \frac{1}{2\tilde{N}(\xi) + 4(1 - \ln 2)} \quad . \quad (25)$$

and use initial condition Eq. (15) to get

$$\tilde{N}(0) = 2 - 4(1 - \ln 2) = 0.39 \quad , \quad (26)$$

and

$$\xi_0 = -2\tilde{N}(0) - 4(1 - \ln 2) \ln \tilde{N}(0) = 0.40 \quad . \quad (27)$$

By Eq. (24), the asymptotic of the amplitude in the saturation region is

$$\tilde{N}(\xi) = \frac{1}{2}\xi \quad . \quad (28)$$

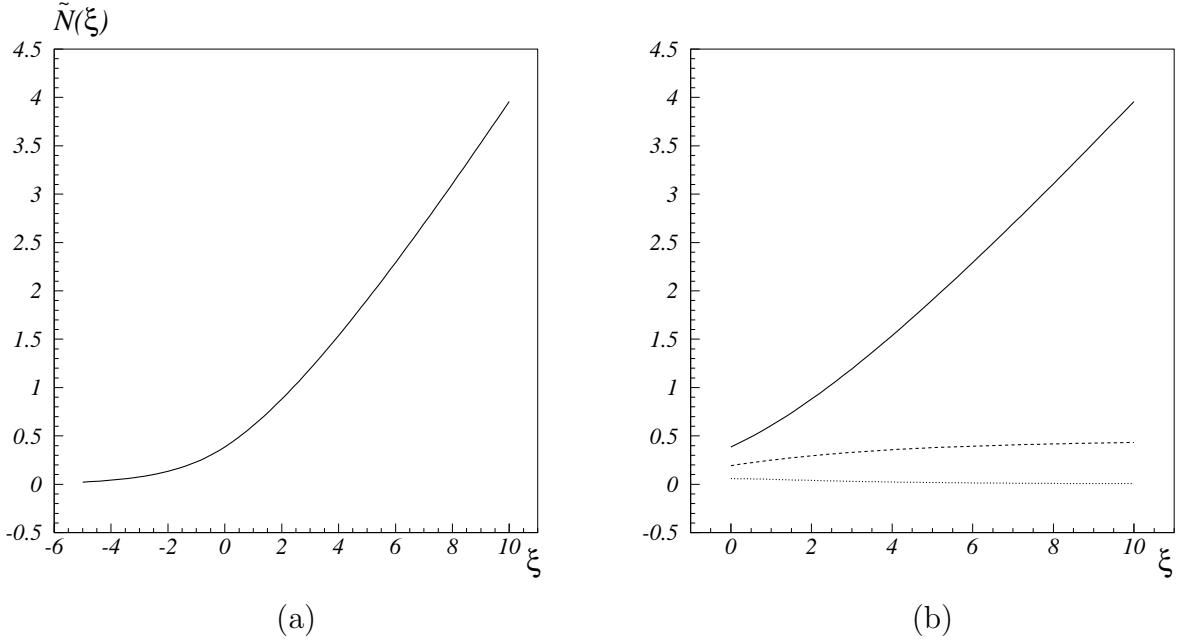


Figure 2: (a) The scaling solution of the Eq. (17) in our model. At $\xi > 0$ this is a numerical solution of Eq. (24) while at $\xi < 0$ of Eq. (29). (b) The scaling solution (solid line), it's first (dashed line) and second (dotted line) derivatives.

In the region $\xi < 0$ we employ the diffusion approximation Eq. (22). The scaling equation then reads

$$4 \frac{d}{d\xi} \tilde{N}(\xi) = 4 \ln 2 \tilde{N}(\xi) + 14\zeta(3) \left(\frac{d}{d\xi} - \frac{1}{2} \right)^2 \tilde{N}(\xi) - \tilde{N}(\xi)^2 . \quad (29)$$

Initial conditions for this equation are Eq. (15) and obvious requirement of the continuity $\tilde{N}(\xi \rightarrow -0) = \tilde{N}(\xi \rightarrow +0)$.

We show the numerical solution to the scaling equation in the whole kinematical region in Fig. 2 (a). In Eq. (21) we neglected second and higher derivatives of the amplitude in comparison to the first one and the amplitude itself. In the Fig. 2 (b) it is shown that the neglection of the higher derivatives was justified. Moreover, the statement that the forward scattering amplitude is slowly varying function holds in general, regardless of model, as scaling variable becomes positive and large.

4 Corrections to the scaling solution.

Now, as we know the scaling solution of Eq. (5) at $\xi \geq 0$, let us find correction to this solution at $\xi < 0$ due to the deviation from the scaling. Substituting Eq. (13) into the Eq. (17), employing diffusion approximation Eq. (22) and keeping terms linear in perturbation $\delta\tilde{N}(\xi, y)$ we arrive at

$$\begin{aligned} -\frac{1}{\bar{\alpha}_s} \frac{\partial}{\partial y} \delta\tilde{N}(\xi, y; b_t) + 14\zeta(3) \frac{\partial^2}{\partial \xi^2} \delta\tilde{N}(\xi, y; b_t) - (4 + 14\zeta(3)) \frac{\partial}{\partial \xi} \delta\tilde{N}(\xi, y; b_t) \\ + (4 \ln 2 + \frac{7}{2}\zeta(3) - 2\tilde{N}(\xi)) \delta\tilde{N}(\xi, y; b_t) = 0 , \end{aligned} \quad (30)$$

where $\tilde{N}(\xi)$ is the scaling solution at $\xi < 0$. Let us define Melin transform $\delta\tilde{N}(\xi, \mu; b_t)$ of the scaling-violating correction $\delta\tilde{N}(\xi, y; b_t)$ with respect to the variable $\bar{\alpha}_s y$

$$\delta\tilde{N}(\xi, y; b_t) = \int_{a-i\infty}^{a+i\infty} \frac{d\mu}{2\pi i} e^{\mu\bar{\alpha}_s y} \delta\tilde{N}(\xi, \mu; b_t) \quad , \quad (31)$$

where a is situated to the right of all singularities of the integrand. Employing Melin transform one rewrites Eq. (30) in the following form:

$$\begin{aligned} 14\zeta(3) \frac{\partial^2}{\partial\xi^2} \delta\tilde{N}(\xi, \mu; b_t) - (4 + 14\zeta(3)) \frac{\partial}{\partial\xi} \delta\tilde{N}(\xi, \mu; b_t) \\ + (4\ln 2 + \frac{7}{2}\zeta(3) - 2\tilde{N}(\xi) - \mu) \delta\tilde{N}(\xi, \mu, b_t) = 0 \quad . \end{aligned} \quad (32)$$

The boundary condition to this equation is specified by Eq. (14). We have to show, however, that this boundary condition does not contradict the solution at $\xi > 0$, i.e. the solution of the Eq. (30) for the correction $\delta\tilde{N}(\xi, y)$ is small in the region $\xi \geq 0$. Using Eq. (21) we get by analogy with Eq. (32)

$$\frac{\partial}{\partial\xi} \delta\tilde{N}(\xi, \mu; b_t) (\mu + 4(1 - \ln 2) + 2\tilde{N}(\xi)) = \delta\tilde{N}(\xi, \mu; b_t) (1 - 2\tilde{N}'(\xi)) \quad . \quad (33)$$

$\tilde{N}'(\xi)$ quickly approaches $\frac{1}{2}$ as ξ increases, so, indeed neglection of $\delta\tilde{N}(\xi, y; b_t)$ at $\xi \geq 0$ is justified.

Returning back to Eq. (32) we see, that non-linear term can be neglected in the first approximation since $4\ln 2 + \frac{7}{2}\zeta(3) \gg 2\tilde{N}(\xi)$ (see Fig. 2 (a)). Thus, we obtain the following solution to Eq. (30):

$$\delta\tilde{N}(\xi, \mu, b_t) = e^{(\frac{1}{2} + \frac{1}{7\zeta(3)})\xi} \left(C_1(\mu, b_t) e^{\frac{\sqrt{\nu(\mu)}\xi}{7\zeta(3)}} + C_2(\mu, b_t) e^{-\frac{\sqrt{\nu}\xi}{7\zeta(3)}} \right) \quad , \quad (34)$$

where $C_1(\mu, b_t)$ and $C_2(\mu, b_t)$ have to be chosen to satisfy the boundary condition Eq. (14)

$$C_1(\mu, b_t) = \bar{\alpha}_s \quad , \quad C_2(\mu, b_t) = 0 \quad , \quad (35)$$

and we introduced notation

$$\nu(\mu) = 7\zeta(3) + 1 - 14\zeta(3)\ln 2 + \frac{7}{2}\zeta(3)\mu \quad . \quad (36)$$

Using Eq. (31) we obtain the final expression for the scaling-violating correction

$$\delta\tilde{N}(\xi, y; b_t) = \frac{|\xi|}{\sqrt{\bar{\alpha}_s 8\pi 7\zeta(3)y^3}} e^{\bar{\alpha}_s y(4\ln 2 - 2 - \frac{2}{7\zeta(3)})} e^{(\frac{1}{2} + \frac{1}{7\zeta(3)})\xi} e^{-\frac{\xi^2}{8\bar{\alpha}_s 7\zeta(3)y}} \quad . \quad (37)$$

To make the b_t dependence of the correction manifest we rewrite it in (y, k^2) coordinates

$$\delta\tilde{N}(\xi, y; b_t) = \frac{|4\bar{\alpha}_s y - \ln k^2 - \beta(b_t, A)|}{\sqrt{\bar{\alpha}_s 8\pi 7\zeta(3)y^3}} e^{4\ln 2\bar{\alpha}_s y - \frac{1}{2}\ln k^2 - \frac{\ln^2 k^2}{56\zeta(3)\bar{\alpha}_s y}} e^{-\frac{\beta}{2} \left(1 + \frac{4\ln k^2}{56\zeta(3)\bar{\alpha}_s y} \right) - \frac{\beta^2}{56\zeta(3)\bar{\alpha}_s y}} \quad . \quad (38)$$

In the limit $\ln k^2 \ll \alpha_s y$ it coincides with the solution to the BFKL equation in the diffusion approximation as it must be since we neglected the non-linear term in Eq. (30).

The numerical value of the ratio $\delta\tilde{N}(y, \xi)/\tilde{N}(\xi)$ is shown in Fig. 3. We see that there is a wide kinematical region where $\delta\tilde{N}(\xi, y) \ll \tilde{N}(\xi)$. Correction increases in the following kinematical regions: $y \rightarrow \infty$ and $\xi \rightarrow -\infty$. Increasing of the correction at $y \rightarrow 0$ is merely an artifact of the boundary condition Eq. (14).

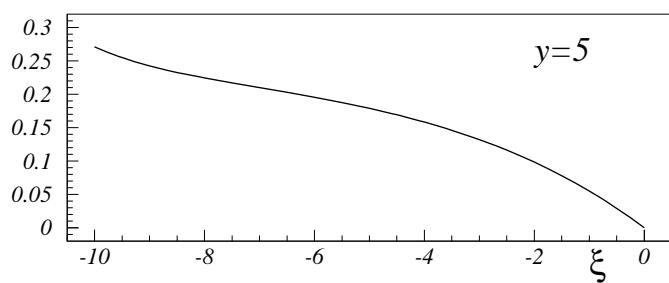
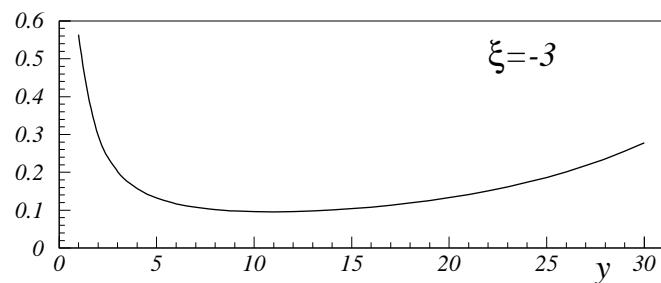
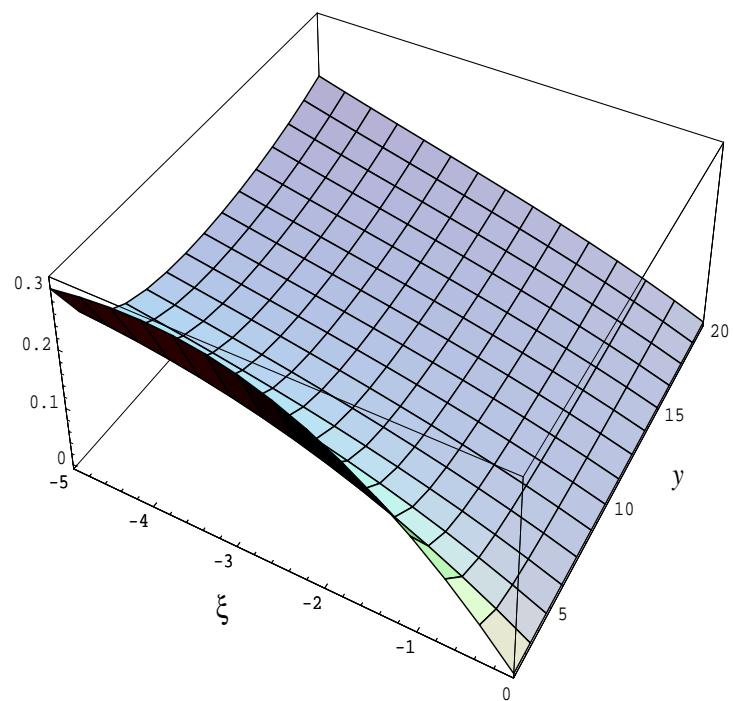


Figure 3: The ratio $\delta\tilde{N}(y, \xi)/\tilde{N}(\xi)$.

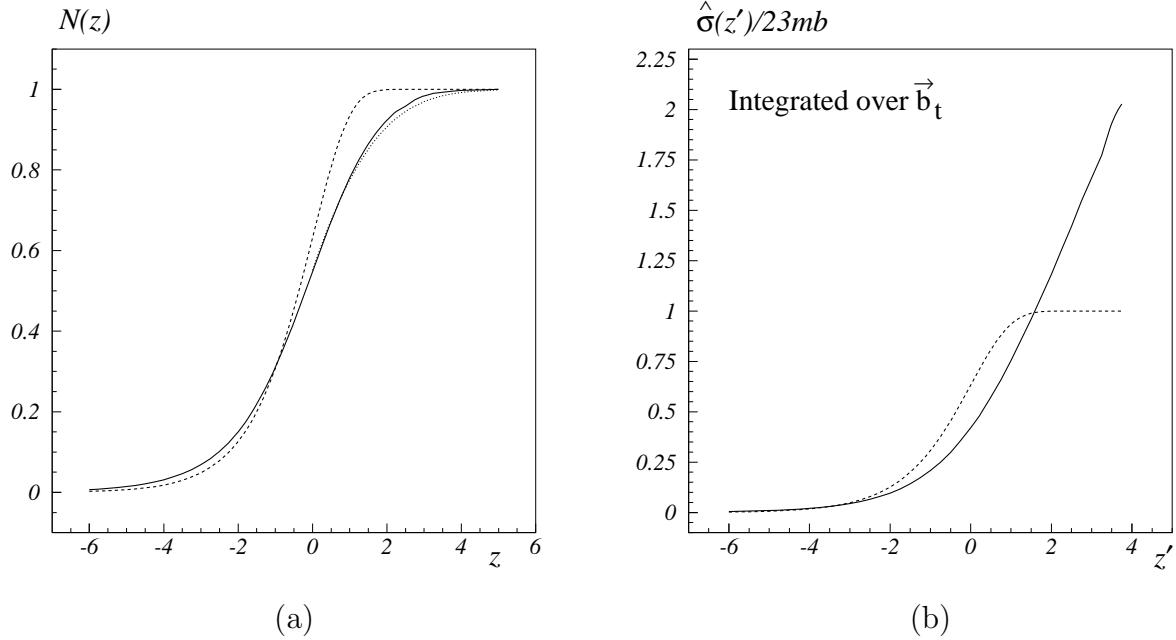


Figure 4: (a) Dipole - target scattering amplitude $N(z)$ and (b) dipole - target cross section $\hat{\sigma}(z')$ in the scaling approximation versus scaling variable (a) z and (b) z' : Solid line is a Fourier transform of $\tilde{N}(\xi)$ given in Fig. 2(a), dashed line is a Golec-Biernat - Wusthoff model as explained in text and dotted line is the $z \gg 1$ asymptotic calculated in Ref. [18].

5 Discussion.

In the previous two sections we have shown that in the wide kinematical region $x_B < x_B^0$ the dipole - target amplitude $\tilde{N}(k, y)$ is a function of one variable ξ . For practical uses we need to Fourier transform the amplitude to the dipole-configuration space. Using Eq. (3) one obtains

$$N(z) = e^z \int_0^\infty dt t J_0(e^{z/2} t) \tilde{N}(-\ln t^2) \quad , \quad (39)$$

where we used Eq. (12), then introduced a new integration variable $t = \exp(-\xi/2)$ and defined

$$z = \ln \frac{\vec{x}^2}{\vec{x}_0^2} + 4\bar{\alpha}_s y - \beta(b_t, A) \quad (40)$$

which is a dipole-configuration space scaling variable. The result of numerical calculation is shown in Fig. 4 as well as the result of our previous paper [18] where we found the asymptotic $z \gg 1$ solution to the evolution equation. The dipole cross section is introduced according to

$$\hat{\sigma}(z') = 2 \int d^2 \vec{b}_t N(z(\vec{x}, y, b_t)) \quad , \quad (41)$$

where $z' = \ln \vec{x}^2 / \vec{x}_0^2 + 4\bar{\alpha}_s y$.

Let us compare the results of our calculation with the successful phenomenological model (for $A = 1$) proposed by Golec-Biernat and Wusthoff[21]. Note, that they assumed that the b_t -dependence of the amplitude factorizes out in the following way:

$$N(z)_{GBW} = N(z')_{GBW} \cdot \theta(b_{t0}^2 - b_t^2) \quad , \quad (42)$$

where $2\pi b_{t0}^2 = 2\pi R_p^2 = 23\text{mb}$. We plotted $N(z)_{GBW}$ in the Fig. 4(a) and $\hat{\sigma}(z')_{GBW}$ in the Fig. 4b (dashed curves). It is seen that while the amplitudes in the Fig. 4(a) are quite close, the dipole cross section differ significantly as z' becomes positive and large. This difference is originated in b_t integration and reflects the fact that our typical b_t^2 is of the order of $R^2 \ln(Q_S^2(x_B)/Q^2)$ [18]. Since in current experiments the shadowing corrections are still small, the closeness of the dashed and solid curves in Fig. 4(b) at $z' < 0$ explains why such over-simplified model managed to describe the experimental data well. However, in future experiments we will enter the region of $z > 0$ where the model of Golec-Biernat and Wusthoff does not work.

We understand the nature of the “geometric” scaling phenomenon noticed in [20]. While in the kinematical region of large y and small k^2 (i.e. $z < 0$) this scaling solution is an exact solution of the evolution equation, at $z > 0$ the scaling holds approximately. We see in Fig. 3 that corrections to the scaling behaviour are small in wide kinematical region.

In conclusion, we would like to emphasize that our approach is developed for rapidities much larger then $y_A \sim \ln A^{1/3}$. In the forthcoming publication we are going to consider the “geometric” scaling in DIS on heavy nuclei including rapidities $y \sim y_A$.

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